Performance Based Learning and Assessment Task

Surface Area of Boxes

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

In the Surface Area of Boxes activity, students will first discuss what surface area is and what they already know about it. Then, with a partner, students will explore and find the surface area of cardboard boxes by using their knowledge of area and tracing nets of the boxes to find the surface area. Surface area is the amount of space covering a 3-D object that is specific to the dimensions and shape of the object. Surface area changes with these dimensions and can be measured. Students must understand surface area for many home improvement projects, such as painting, putting up wallpaper, reupholstering furniture, et cetera

II. UNIT AUTHOR:

Maggie Hughes, Hidden Valley High School, Roanoke County Public Schools.

III. COURSE:

Geometry

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

Students will be able to: 1) find surface area of any cube or rectangular prism. 2) find surface area of other three-dimensional objects, such as a cone, pyramid, or sphere. 3)measure, make observations, analyze, and describe surface area and changes made to the dimensions that affect it. 4) develop meaningful representations, both as formulas and figures, of surface area

VI. REFERENCE/RESOURCE MATERIALS:

Students will need: 3x3 Graphic Organizer, Rulers, Clean, small boxes, Attached Boxes Activity worksheet, Butcher (or large-format) paper (only enough to cover one box per set of partners--this would be pre-determined by teacher when they work through the lesson prior to teaching it), Classroom set of calculators, Attached Assessment & Rubric Sheets.

VII. PRIMARY ASSESSMENT STRATEGIES:

Attached is the Assessment List and Rubric for the Surface Area of Boxes Activity. Both focus on the student's successful completion of the activity, how they've presented their work, and if they are able to discuss and explain what they've done

VIII. EVALUATION CRITERIA:

Students will be evaluated on their participation in the introductory discussion and 3x3s activity, accuracy and completion of the activity and worksheet, completion of their homework, and their discussion with the instructor after they have turned in the activity and homework. The student's participation in the introductory discussion will be evaluated using an observation checklist.

IX. INSTRUCTIONAL TIME:

One ninety-minute class session

Surface Area of Boxes

Strand
Geometry

Mathematical Objective(s)
Students will be able to: 1) find surface area of any cube or rectangular prism. 2) find surface area of other three-dimensional objects, such as a cone, pyramid, or sphere. 3)measure, make observations, analyze, and describe surface area and changes made to the dimensions that affect it. 4) develop meaningful representations, both as formulas and figures, of surface area

The mathematical goal of this activity is for students to understand and explain the concept of surface area and how to find it. Students are exploring surface area through a hands-on activity in which they measure surface area of a box. This box could represent a room they're putting wallpaper in, a house they are painting, or a cake they are icing. Students are using logic and reasoning to create formulas after they've discussed and discovered how surface area is found without a mathematical formula.

Related SOL

- G. 13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- G. 14 The student will use similar geometric objects in two- or three-dimensions to b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;d) solve real-world problems about similar geometric objects.

NCTM Standards

- Analyze properties and determine attributes of three-dimensional objects.
- Explore relationships among three-dimensional objects.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects from different perspective
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest.
- Make decisions about units and scales that are appropriate.
- Understand and use formulas for the surface area of geometric figures.
- Organize their mathematical thinking through discussion with peers.
- Communicate their thinking clearly to teacher and peers.
- Analyze and evaluate the mathematical thinking and strategies of their partners.
- Use the language of mathematics to express mathematical ideas precisely.

- Create and use representations to record and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations.
- Use representations to model and interpret physical and mathematical phenomena.

Materials/Resources

Students will need: 3x3 Graphic Organizer, Rulers, Clean, small boxes, Attached Boxes Activity worksheet, Butcher (or large-format) paper (only enough to cover one box per set of partners--this would be pre-determined by teacher when they work through the lesson prior to teaching it), Classroom set of calculators, Attached Assessment & Rubric Sheets.

Assumption of Prior Knowledge

- Students should have an understanding of area of 2-dimensional objects, how to find it, and be able to explain how it is found. They should also have an idea of what surface area is, for example, know that it applies to 3-dimensional objects and covers the entire area of the object.
- Student should be operating on Level 3 Abstraction on the Van Hiele scale with respect to 3- dimensional figures. They should also be operating on Level 3 Abstraction of Level 4 Deduction with respect to area of 2-dimensional figures.
- Students will begin to express ideas of surface area being the total area of each face of an object, formulas of surface area for rectangular prisms, and how changing the dimensions of a rectangular prism change the areas of the faces and thus the surface area of the object. They may not use the exact formulas that are typically provided but they will describe how they found the surface area and attempt to explain a formula verbally, with sentences, or with words instead of variables.
- Students should have already explored and discussed area of two-dimensional figures and three-dimensional figures (spheres, rectangular prisms, cylinders, et cetera).
- Students can discuss home improvement projects; baking; creating skins and covers for cell phones, laptops, or tablets. They may also have studied cellular respiration in Biology classes. If so, students are able to discuss the impact of surface area of cells on their respiration rates.

Introduction: Setting Up the Mathematical Task

Write Agenda on
board: Agenda
1. Questions?
2. Surface Area 3x3s *(approximately 25 minutes)*
3. Boxes Activity and Worksheet *(approximately 50 minutes)*

Discuss the agenda with the students. “In this activity, we will discuss the surface area of three-dimensional objects. First we will create 3x3 graphic organizers to help see what we know and

don't know about surface area. We will develop these 3x3s and discuss the terms and our findings. Next, we will develop ideas and formulas about how to find the surface area of a rectangular prism by covering boxes with construction paper. Within the activity, you will be answering questions on a guided worksheet to help develop your understanding of surface area."

As an entire class, ask students what they think surface area is and where it exists. Is surface area on all three-dimensional objects? How do you know? Follow up discussion with brief explanation of what surface area truly is (use statements that students provide and build on them).

Classroom Discussion and Activity: Surface Area 3x3s *(approximately 25 minutes*
Draw 3x3 on board (see example on page 4)

- Place "Surface Area" in center box.
- Allow students to come up with relevant terms, formulas, etc. to fill in other 8 boxes.
- Ask students to pick the two most important terms from the 3x3 to create two more3x3s with the two important terms in the center of those. (Can do one relevant term and one more 3x3, if needed, due to time or if students seem to be understanding or getting bored.)
- Allow students to come up with more relevant terms, etc. to fill in the new 3x3s. Either fill in the 3x3s yourself or have a student volunteer to scribe.

If students struggle, ask them what they know about "normal" area of two-dimensional objects. What's the difference between two-dimensional and three-dimensional objects? What do you know about each? Briefly discuss area again, if necessary, to get their ideas flowing. Ask students what this activity shows them. Are there any terms they do not understand or need more explanation on? Where are the connections between these terms,
formulas, etc? Are they all interconnected and related? Make sure students see that surface area has many connections and is not just a formula applied to a word problem or picture.

Students will discuss the 3x3s as an entire class. The instructor will create an observation checklist of who contributes to the discussion or asks relevant questions to monitor their participation.

Student Exploration

Student/Teacher Actions

Have students to pair up with a partner that is at their table. Seating at tables will be assigned by instructor. Hand out two boxes per each group (or one box per student). Ask students how these boxes are applicable to surface area. Do they realize that every box has surface area? How do you know? What is the mathematical name for a "box"?

(Hopefully, students realize that surface area is the area of each of the surfaces of the boxes. Students should also be able to state that these boxes are mathematically called rectangular prisms.)

Pass out 1 worksheet *(see attached)* per student. Explain that they will be exploring the surface area of their boxes and allow them to skim the worksheet and ask any questions. Inform students that they are to record all work and final answers on the butcher paper and the worksheet.

Talk with students about how polished final work is important so that anyone can understand the processes they use to reach their conclusions. This work could be used to have students defend there knowledge and when developing proofs. It will also be used as a formative assessment.

Once students begin working on worksheets and activity, walk around to each table/partners and make sure students stay on track. Answer any questions they may have, give hints to keep their thinking going. Teachers should be asking students to explain what they are doing, especially to uncover any misconception the teacher observes. The teacher is to be as hands-off in giving answers as possible. The goal is for the discovery activity to be student-centered.

To integrate technology, students could use Geometer's Sketchpad or GeoGebra to create replications of their boxes. They could explore the boxes on these applications to develop formulas and ideas about surface area. Additionally, when students are asked to morph their boxes, these applications would give them a visual of what they are doing when they change the dimensions of their boxes.

Monitoring Student Responses

Students are expected to communicate their thinking and new knowledge by writing down their findings, creating paper-covered boxes, explaining their findings to their partner and to the teacher.

Students will effectively communicate to their partner by equally participating in the activity and discussing what they are doing at each step. They are to ask each other questions and try to develop ideas as a team. They can also record their findings on the butcher paper to aid in comparing and contrasting between the students' boxes.

The teacher will introduce each activity and how it applies to surface area. Additionally, the teacher will facilitate the understanding of these ideas by giving hints and asking probing questions to the partner sets as they complete the activity. There is also a summary discussion, homework, and meetings with students to help clarify the concept of surface area.

For students that are ready to move forward, the teacher will have additional questions about the surface area of other objects. What about spheres or cylinders that do not have flat faces? Students could explore and develop these ideas on Geometer's Sketchpad or GeoGebra.

Make sure students know to do the homework. Read the instructions of the homework as a class and answer any questions that students may have. Give the following link to students to help them understanding how changing their boxes would change the surface area: http://www.geogebra.org/en/upload/files/english/lewisprisco/rectangular_prism.html
Make sure students know that they will be meeting with you to discuss their findings from the activity and the homework. Let them know that they will need to be able to meaningfully explain their findings and ideas. The teacher will use their box, activity worksheet, homework, and discussions as a way to assess the students' knowledge of the surface area of rectangular prisms, specifically.
Talk to students about their boxes from the activity. Were they able to cover them with the amount of paper provided? If they were to paint the box, would they be able to figure out how much paint they needed if they knew the surface area?
Where, outside of the classroom, would surface area also need to be applied? What about painting their house? Or figuring out how many windows would fit on the side of a building? How about icing a cake?

Assessment List and Benchmarks

Assessment List, Rubric, and Benchmarks are attached.
Assessments for each task and activity are provided within the Introduction, Activity, and Summary of the described task. Additional guidance in the activity is shown in the Surface Area of Boxes Activity Worksheet.

Surface Area of Boxes Activity Worksheet

Directions:

1. Use the two boxes that you and your partner have been provided to answer the following questions and fill in the tables. Please follow specific instructions within each question.

    1. Determine which box is Box 1 and which is Box 2.
    2. Label the sides of the boxes (Top, Bottom, Right Side, Left Side, Front, Back).
    3. Write the type of box beside Box 1 or Box 2.
    4. Measure the area of each side of each of the boxes using the ruler and the formula you know for area. Use centimeters and be sure to include units! Write the information below.

BOX 1:

Formula used to find area: _____

Area of:

Top: _____

Bottom: _____

Right Side: _____

Left Side: _____

Front: _____

Back: _____

TOTAL: _____

Surface Area = _____

BOX 2:

Formula used to find area: _____

Area of:

Top: _____

Bottom: _____

Right Side: _____

Left Side: _____

Front: _____

Back: _____

TOTAL: _____

Surface Area = _____

2. Trace a net of the box onto the butcher paper. Indicate edges/fold lines with dotted lines. Label each side as Top, Bottom, Right Side, Left Side, Front, or Back, corresponding to how you've labeled the actual box.
3. Measure the net and write the dimensions of each side on the inside of the traced net. Write the area of each side in the net drawing, as well.
4. Using these dimensions, figure out what the surface area would be if the net drawn on the paper created a rectangular prism. Does the surface area match that of the box you traced?
5. Cut out the net that you've drawn. Wrap it around the box and tape or glue it into place. Does it fit the box? Should it? How should you have known, before you wrapped the box with the net, that it was going to fit? What measurements would show this?
6. Now, pretend Box 1 has morphed and one of the sides is halved. Recreate the table that matches whichever Box 1. Does the surface area change? Is it halved? How and why? Do the same process and answer these same questions for a doubled side on Box 2. If you need help, use the following link: http://www.geogebra.org/en/upload/files/english/lewisprisco/rectangular_prism.html

7. Develop a formula that could be used to figure out the ENTIRE surface area of your box, not the area of just one of the sides. Why is the total of all areas of the sides called surface area instead of just area?
8. Imagine that the rectangular prism is a door. What happens to the surface area if we cut hole out for a mail slot or doorknob? How would you modify your formula for surface area?

Homework:

At your house, find another object, other than a cube or rectangular prism, to find the surface area of. It may also be a combination of three-dimensional objects. You may use outside resources to figure out how to find the surface area. Create a chart to show how you found the surface area. If possible, bring in the object that you found the surface area of or take a picture of it from all perspectives (see me for more details). Draw a three-dimensional, to-scale picture of the item you explored. Record all results and draw a picture representing your figure. Save your drawing and results to bring into class.

*Bonus Options:

Draw a to-scale representation of your object in Geometer's Sketchpad, save the file, and submit via Blackboard (or whichever system the school uses or email).

Present your object, pictures, and/or drawings to the classroom. What type of three-dimensional object did you explore? Discuss the object and the methods you used to find the surface area of the object. Please see me to work out details and visual representations used in your presentation.

Benchmark

Due to the nature of the activity, only one box has been provided as a benchmark. Sets of partners would need to turn in two sets of this information since they are graded as a team. Since all work is submitted electronically, I've only provided images of what the box could look like. A covered box is one that has just been neatly covered in butcher paper with labels, dimensions, and each side's area. The faces of the rectangular prism (or box) will have butcher paper covering the sides, with no overhang or part of the box showing.

Surface Area of Boxes Activity Worksheet

Directions:

Directions:

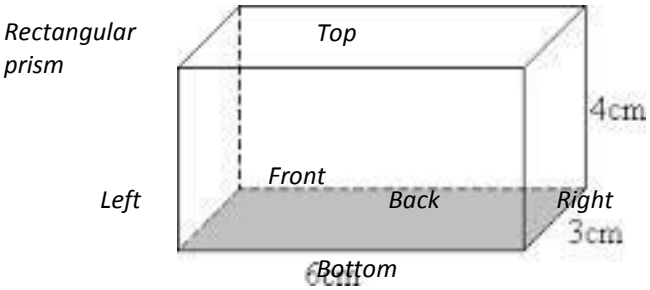
1. Use the two boxes that you and your partner have been provided to answer the following questions and fill in the tables. Please follow specific instructions within each question.

    1. Determine which box is Box 1 and which is Box 2.
    2. Label the sides of the boxes (Top, Bottom, Right Side, Left Side, Front, Back).
    3. Write the type of box beside Box 1 or Box 2.
    4. Measure the area of each side of each of the boxes using the ruler and the formula you know for area. Use centimeters and be sure to include units! Write the information below.

BOX 1:
Formula used to find area: A = *lw*

Area of:

- Top: *18 cm²*
- Bottom: *18 cm²*
- Right Side: *12 cm²*
- Left Side: *12 cm²*
- Front: *24 cm²*
- Back: *24 cm²*
- TOTAL: *108 cm²*



Surface Area = *108 cm²*

1. Use the scissors, construction paper, glue, and areas to cover the boxes (one side at a time) with different colored construction paper. Be sure to measure the dimensions of each side and record them in a chart, similar to the one above, before covering the box. Use length, width, and height in your chart.
   Top and Bottom: 6cm x 3cm
   Left side and Right side: 3cm x 4cm
   Front and Back: 6cm x 4cm

2. Trace a net of the box onto the butcher paper. Indicate edges/fold lines with dotted lines. Label each side as Top, Bottom, Right Side, Left Side, Front, or Back, corresponding to how you've labeled the actual box.
3. Measure the net and write the dimensions of each side on the inside of the traced net. Write the area of each side in the net drawing, as well.
   Picture about would be representative of this net drawn and folded up to look like the box).
4. Using these dimensions, figure out what the surface area would be if the net drawn on the paper created a rectangular prism. Does the surface area match that of the box you traced?
   Top and Bottom: 18cm^2
   Left and Right sides: 12cm^2
   Front and Back: 24cm^2
   Total surface area: 108cm^2.
   This surface area matches the surface area of the box, without the paper pasted on.

5. Cut out the net that you've drawn. Wrap it around the box and tape or glue it into place. Does it fit the box? Should it? How should you have known, before you wrapped the box with the net, that it was going to fit? What measurements would show this?
   The net fits the box when I match up the edges and fold lines and the sides, accordingly. This net should fit, not only because it was created from this particular box, but because their surface areas and dimensions of each side are the exact same.

6. Now, pretend Box 1 has morphed and one of the sides is halved. Recreate the table that matches whichever Box 1. Does the surface area change? Is it halved? How and why? Do the same process and answer these same questions for a doubled side on Box 2.
   Halved:
   Top and Bottom: 3cm x 3cm – Area = 9cm^2
   Left side and Right side: 3cm x 4cm – Area = 12cm^2
   Front and Back: 3cm x 4cm – Area = 12cm^2
   Surface Area = 66cm^2
   Doubled:
   Top and Bottom: 12cm x 3cm – Area = 36cm^2
   Left side and Right side: 3cm x 4cm – Area = 12cm^2
   Front and Back: 12cm x 4cm – Area = 48cm^2
   Surface Area = 192cm^2
   Surface area does change when a side is halved or doubled. The surface area is not halved or doubled because each edge length is not halved or doubled. If each dimension were halved or doubled, the total surface area would be halved or doubled, respectively. The dimensions of the box are used to determine the area of each side, thus changing the total surface area when each side's area is changed.

7. Develop a formula that could be used to figure out the ENTIRE surface area of your box, not the area of just one of the sides. Why is the total of all areas of the sides called surface area instead of just area?
   Surface Area (of a rectangular prism) = 2(top or bottom side) + 2(left or right side) + 2(front or back side)
   Surface Area (of a rectangular prism) = 2lw + 2wh + 2lh, where l is length, w is width, and h is height
   Surface Area covers the entire surface of a three-dimensional figure. Area only applies to a flat, two-dimensional figure that does not have more than one side.

8. Imagine that the rectangular prism is a door. What happens to the surface area if we cut hole out for a mail slot or doorknob? How would you modify your formula for surface area?
   We would need to subtract twice the area of the rectangle of the mail slot or circle for the doorknob. We subtract twice the area to account for the fact that the hole will go through both sides of the door.

Homework:

At your house, find another object, other than a cube or rectangular prism, to find the surface area of. It may also be a combination of three-dimensional objects. You may use outside resources to figure out how to find the surface area. Create a chart to show how you found the surface area. If possible, bring in the object that you found the surface area of or take a picture of it from all perspectives (see me for more details). Draw a three-dimensional, to-scale picture of the item you explored. Record all results and draw a picture representing your figure. Save your drawing and results to bring into class.

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Surface Area of Boxes Activity Assessment List

			Earned Assessment	
#	**Element**	**Point Value**	**Self**	**Teacher**
1	Student participated in 3x3 Discussion by speaking or asking a question at least three times (recorded with an observation checklist by instructor).	2		
2	Student filled in each measurement for each box on worksheet.	2		
3	Student covered each box appropriately with butcher paper.	2		
4	Student recorded measurement findings while completing the activity.	2		
5	Student created a new chart with new "morphed" measurements.	2		
6	Student developed a formula for surface area.	2		
7	Student answered follow-up questions for each instruction on worksheet.	2		
8	Student's Surface Area of Boxes Worksheet is complete.	2		
9	Student's work is well organized.	2		
10	Student's work is neat.	2		
11	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	2		
12	Student can show why their Surface Area formula works for any rectangular prism.	2		
13	Student completed homework.	2		
14	Student meets with teacher to discuss homework and clarify any misconceptions.	2		

Surface Area of Boxes Activity Rubric

#	Element	0	1	2
1	Student participated in 3x3 Discussion by speaking or asking a question at least three times (recorded with an observation checklist by instructor).	Student did not speak or ask any questions.	Student contributed 1 or 2 times.	Student contributed 3 or more times.
2	Student labeled boxes and described on worksheet.	Labels not provided.	Some, but not all, labels provided.	All labels provided for each box.
3	Student covered each box appropriately with butcher paper.	Boxes not covered.	Improper coverage of boxes.	Each face of box is properly covered.
4	Student recorded measurement findings while completing the activity.	Measurements not provided.	Some, but not all, measurements provided.	All measurements provided.
5	Student created a new chart with new "morphed" measurements.	No chart showing morphed box and measurements.	Chart showing some of morphed measurements provided.	Fully completed chart showing morphed measurements.
6	Student developed a formula for surface area.	No formula developed.	Formula developed but does not make sense.	Correct and logical formula developed.
7	Student answered follow-up questions for each instruction on worksheet.	No follow-up questions answered.	Follow-up question answered but are not correct or do not make sense.	Follow-up questions answered correctly or very close to being correct.
8	Student's Surface Area of Boxes Worksheet is complete.	Incomplete worksheet.	Some of worksheet completed.	Entire worksheet completed.
9	Student's work is well organized.	No evidence of organization.	Not fully organized.	Well-organized work.
10	Student's work is neat.	Lacks neatness.	Needs improvement.	Neat and legible.
11	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	Student provides no explanation.	Explanation provided with logical flow, is mostly incorrect.	Explanation provided with logical flow and is mostly correct.
12	Student can show why their Surface Area formula works for any rectangular prism.	Cannot show how formula works.	Partially shows why formula works.	Correctly and completely shows how formula works.
13	Student completed homework.	Incomplete	Partially complete	Fully complete
14	Student meets with teacher to discuss homework and clarify any misconceptions.	No meeting held.	Student does not contribute to meeting.	Student contributes to meeting.